A fuzzy approach of online reliability modeling and estimation

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Abstract

In maintenance field, traditional concepts like preventive and corrective strategies are progressively completed by new ones like predictive and proactive maintenance. For that purpose, a fundamental task is the estimation of the provisional reliability of an equipment as well as its remaining useful life. However, traditional approach of reliability based on statistical analysis can be not suitable as very few knowledge can be available. Within this frame, the general purpose of the work is to explore the way of developing a fuzzy approach of on-line reliability modeling and estimation in order to take into account the uncertainty as well as possible. A federative point of view of the reliability modeling process and of the prognostic of degradation activity is proposed. From that, two ways of considering uncertainty in reliability modeling are discussed (probabilistic, fuzzy/possibility approaches), and the inherent limits of both methods are pointed out.

Keywords: Reliability, prognostic, degradation, fuzzy logic, possibility theory.

1. Introduction

The growth of reliability, availability and safety of a system is a determining factor in regard with the effectiveness of industrial performance. As a consequence, the implementation of provisional maintenance strategies is a good way to improve the availability of processes, to ensure the smallest variations of products qualities or the direct costs falling. For that purpose, a fundamental task is the estimation of the provisional reliability of an equipment as well as its remaining useful life. However, in practice, this can be very difficult to achieve: real systems are complex; there are many uncertainties upon their behaviors. Moreover, traditional approach based on statistical analysis can be not suitable as very few knowledge can be available. Within this frame, the general purpose of the work is to explore the way of developing a fuzzy approach of on-line reliability modeling and estimation in order to take into account the uncertainty as well as possible. Thereby, different ways of considering uncertainty in reliability modeling are discussed and the inherent limits of all methods are pointed out.

The paper is organized in two main parts.
The first part is dedicated to the problem's statement. Fundaments of reliability are given and the principles of degradation modeling are also presented. As a global point of view, the on-line provisional reliability estimation is compared to the "prognostic" process: two global tasks must be ensured, a first one to "predict" the evolution of a situation, and a second one to "assess" this predicted situation with regards to an evaluation referential. According to it, potentials informational frameworks in reliability modeling are studied in the second part. Indeed, the traditional reliability theory based on a probabilistic formalization of the failure mechanisms can be critiqued since the available information may be insufficient. Thus, two ways of considering uncertainty in reliability modeling are discussed (probabilistic, fuzzy/possibility approaches), and the inherent limits of both methods are pointed out.

2. Problem's statement

2.1 Fundaments of reliability theory

According to International Committees for Standardization, reliability (R) is defined as "the ability of an item to perform a required function under given conditions for a given time interval" [1, 2]. The term "reliability" is also used as a measure of performance and may be defined as a probability: reliability ($R(t)$) is the probability that a failure does not occur before time t. If the random variable ϑ denotes the time to failure, $f_\vartheta(t) = Pr(\vartheta = t)$ its probability distribution function (in reliability work, it is known as the failure density function), and $F_\vartheta(t) = Pr(\vartheta \le t)$ its cumulative distribution function, then the reliability is defined as proposed in eq. (1).

$$R(t) = 1 - F_\vartheta(t) = 1 - \int_0^t f_\vartheta(u).du \qquad (1)$$

In practice, aggregated dependability measures like the mean time between failures ($MTBF$) and the mean time to failure ($MTTF$) can be preferred to manage the preventive maintenance policies optimization. Indeed, such indicators enable evaluating the average time before the unexpected failure event. $MTTF$ is defined as:

$$MTTF = \int_0^\infty R(t).dt \qquad (2)$$

The mathematical fundaments of reliability can be extended to the case of multiple failures modes. This is not presented in this paper for clarity but more details can be found in [3].

2.2 Reliability estimation by using degradation modeling

The mathematic of reliability proposed here above supposes that failure can be characterized by a random variable. This can be difficult to obtain and another way of formalizing the reliability is that of degradation modeling.

Let assume now that the failure is characterized by the fact that the degradation of the equipment (y) overpass a degradation limit (y_{lim}). At any time t, the failure probability can thereby be defined as follows:

$$F_{fail}(t) = Pr\left[y(t) \geq y_{lim}\right] \tag{3}$$

Assuming that the degradation can be probabilistically modeled (Figure 1), let note $g_{y/t}(t)$ its probability distribution function at time t. Thereby, by analogy with reliability theory, the reliability modeling is, at time t:

$$R(t) = 1 - F_{fail}(t) = 1 - Pr\left[y(t) \geq y_{lim}\right] = 1 - \int_{y_{lim}}^{\infty} g_{y/t}(u).du \tag{4}$$

The remaining time to failure (TTF) of the system can finally be expressed as the remaining time between current time (tc) and the time to underpass a reliability limit (R_{lim}) fixed by the practitioner (Figure 1). This can be generalized with a multi-dimensional degradation signal. See [4] or [5] for more details.

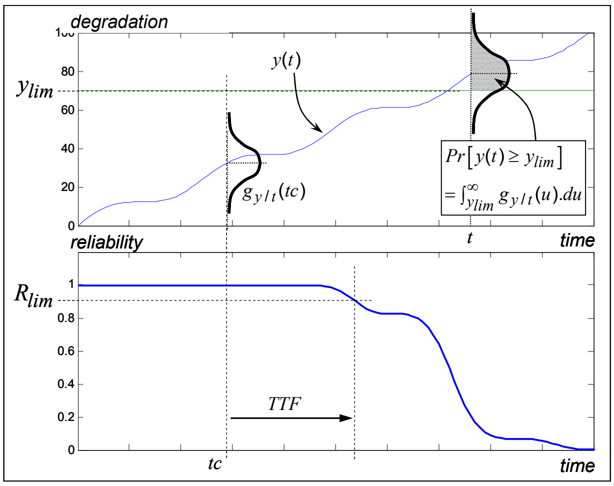

Figure 1. Degradation and reliability modeling.

2.3 Limits, links with prognostic and problems inherent to uncertainty

The above statements reveal that a key issue in reliability prediction is the apprehension of the failure mechanism.

In traditional approach, the failure distribution is either obtained empirically by evaluating the ratio of items that do not perform their function in a stated period to the total number in the sample, or expressed by an expert. In degradation modeling approach, the same method can be conducted to construct a degradation model $y(t)$. However, if failures observations are rare, these approaches can be difficult to achieve, even impossible. Indeed, industrial not always can test several systems since it can be too expensive. In addition, results depend on the operational conditions and the extrapolation to a specific case can require heavy mathematical treatments. All this is all the more critic as the system evolves and the on-line reliability estimation and prediction are thereby difficult to perform. Reliability modeling can however be assimilated to the prognostic process.

The International Organization for Standardization defines prognostic as "the estimation of time to failure and risk for one or more existing and future failure modes" [2]. Prognostic is also called the "prediction of a system's lifetime" as it is a process whose objective is to predict the remaining useful life (RUL) before a failure occurs given the current machine condition and past operation profile [6]. Thereby, two salient characteristics of prognostic appear (Figure 2 - [7]):

1. prognostic is mostly assimilated to a prediction process (a future situation must be caught). This step of prognostic aims at predicting the degradation signal by tracking the system's evolution,
2. prognostic is based on the failure notion, which implies that it is associated with a degree of acceptability. The assessment process of prognostic aims at evaluating the predicted situation with regards to a referential.

According to this acceptation of prognostic, prognostic and on-line reliability modeling by using degradation modeling are very similar. Following that, two types of uncertainties must be taken into account:

1. this one inherent to the degradation prediction. As there can be very few information about the phenomena under study (and its evolution), probability should be used carefully,
2. this one inherent to referential limits. The acceptability of the degradation limits of an equipment can be unclear and difficult to formalize.

Finally, the reliability modeling is relevant if both types of uncertainties are well bounded and treated in prognostic systems. The purpose of next part is to study the different informational context that can characterize the reliability modeling.

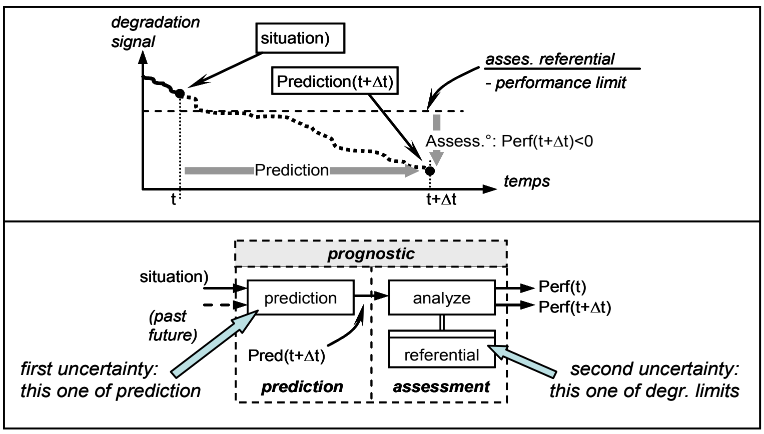

Figure 2. Prognostic process [7] and uncertainties.

3. Towards a fuzzy approach of online reliability modeling

3.1 Probabilistic approach

3.1.1 Principle of the probabilistic modeling approach

Let assume that both the degradation signal and the degradation limit are probabilistically modeled (Figure 3): at time t, the degradation signal y has a probability distribution function $g_{y/t}$, and the degradation limit y_{lim} has a probability distribution function $g_{y_{lim}/t}$ and a cumulative distribution function $G_{y_{lim}/t}$.

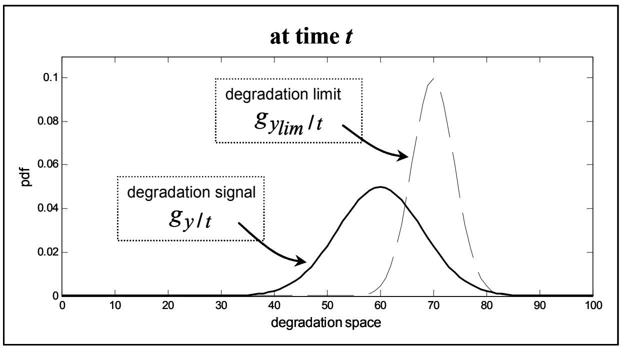

Figure 3. Pdf of the degradation signal and limit at time *t*.

The probability of failure can be expressed as:

$$Pr\left[failure\right] = Pr\left[y_{lim} \le y\right] = \int_0^{\infty} \left(Pr\left[y_{lim} \le u,\ u = y\right]\right).du \quad (5)$$

and, assuming that y and y_{lim} are independent,

$$Pr\left[failure\right] = \int_0^{\infty} \left(Pr\left[y_{lim} \le u\right] \times Pr\left[u = y\right]\right).du \quad (6)$$

Both members of the integral of eq. (6) can be determined as follows:

$$\begin{aligned} &Pr\left[y_{lim} \le u\right] = \int_0^u \left(g_{y_{lim}/t}(x)\right).dx = G_{y_{lim}/t}(u) \\ &Pr\left[u = y\right] = g_{y/t}(u) \end{aligned} \quad (7)$$

and thereby,

$$Pr\left[failure\right] = \int_0^{\infty} \left(G_{y_{lim}/t}(u) \times g_{y/t}(u)\right).du \quad (8)$$

Following that, the reliability modeling expression is at time t:

$$R(t) = 1 - Pr\left[failure\right] = 1 - \int_0^{\infty}\left(G_{y_{lim}/t}(u) \times g_{y/t}(u)\right).du \tag{9}$$

Note that, if the degradation limit is expressed as a simple threshold like in Figure 1, its probability distribution and cumulative functions are:

$$g_{y_{lim}/t}(u) = \begin{cases} 1 \text{ if } u = y_{lim} \\ 0 \text{ elsewhere} \end{cases} \quad ; \quad G_{y_{lim}/t}(u) = \begin{cases} 0 \text{ if } u \in \left[0, y_{lim}\right[\\ 1 \text{ if } u \in \left[y_{lim}, \infty\right[\end{cases} \tag{10}$$

and eq. (9) is thereby simplified: eq. (4) is obtained. This is a particular case of the probabilistic approach.

3.1.2 Illustration and discussion

An illustration of the probabilistic approach is proposed in Figure 4 in which the 3D-graph is that of the probabilistic modeling of the degradation signal an limit, and the 2D-graph, that of the resulting failure and reliability modeling.

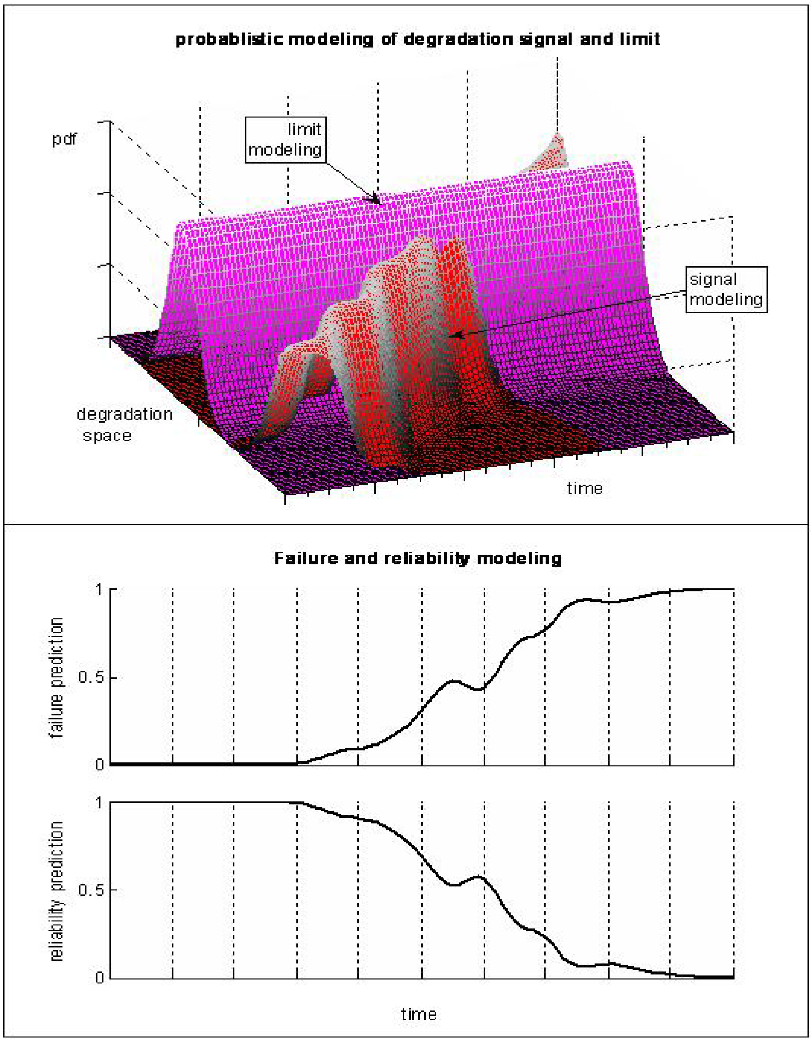

Figure 4. Illustration of the probabilistic approach of reliability modeling.

In practice, this approach can be used if both the degradation signal and the degradation limit are expressed as probability functions. Thus, when performing the prediction step of prognostic, this implies applying statistical techniques in order to take into account the uncertainty of the degradation signal estimation. However, these techniques can be difficult and computationally time expensive to deploy if real systems are complex and non-linear.
With regard to the degradation limit, many experiments on failure occurrence should be used in order to assign the confidence (pdf) on performance threshold. However, if few knowledge of the failure mechanism is available, it can be difficult to formalize it in probabilistic terms. Moreover, translating an expert's knowledge in probabilistic terms injects unjustified information in formalized data.
In order to undergo some limits of probability theory A fuzzy/possibility approach of on-line reliability modeling is proposed in next section.

3.2 Fuzzy / possibilistic approach

3.2.1 Fundaments
Possibility theory was introduced by Professor Lotfi Zadeh in 1978 [8] as an extension of his theory of fuzzy sets and fuzzy logic [9]. D. Dubois and H. Prade further contributed to its development, and, assuming that all principles of fuzzy sets and possibility theory can not be dressed here, more details can be found in [10].

In a few words, fuzzy logic and possibility theory aim at reasoning with imprecise or vagueness knowledge, by introducing a novel way of taking into account uncertainty. Globally, possibility theory enables judging from the veracity of a proposition by the use of two indicators (whereas probability theory that is found on a single measure): the possibility measure (labeled Π) and the necessity measure (labeled N). In order to introduce these measures, let have a look to the possibility distribution concept.

A possibility distribution, labeled π, is an application from the universe of the discourse Ω to the interval [0 , 1]. It characterizes a fact defined on Ω and designates an appreciation on the belonging of all values of Ω to the fact represented. Note that a possibility distribution function is normalized: $\sup[\pi(w)] = 1,\ w \in \Omega$.
Figure 5 shows an example of possibility distribution to formalize that "the occurrence of an event is approximately between 0,2 and 0,3".

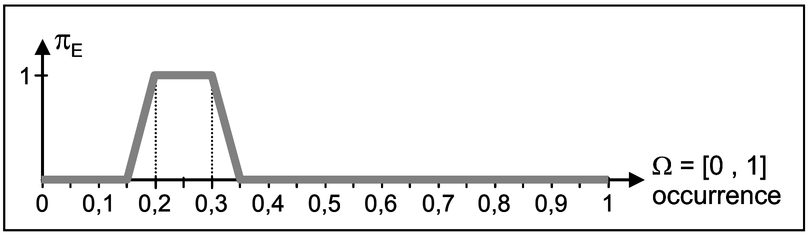

Figure 5. Possibility distribution example: "the occurrence of the event *E* is approximately between 0,2 and 0,3.

Possibility theory introduces confidence measures that allow evaluating the degree with which a fact is in accordance with a reference set and to valuate the degree of certainty of this assertion.

Let note π_F the possibility distribution membership of a fact F, and μ_{Ref} the possibility distribution membership of a reference situation Ref, then, possibility and necessity indicators are:

$$\begin{aligned} \Pi_{F \in Ref} &= \sup_{w \in \Omega} \left\{ min\left[\mu_{Ref}(w), \pi_F(w) \right] \right\} \\ \mathrm{N}_{F \in Ref} &= \inf_{w \in \Omega} \left\{ max\left[\mu_{Ref}(w), 1 - \pi_F(w) \right] \right\} \end{aligned} \tag{11}$$

The possibility measure can be interpreted as the degree of intersection between the values compatibles with Ref and the set of possible values for F and designates thereby the possibility that F corresponds to Ref (scale between 0 and 1). Note, that this measure does not exclude the possibility of the contrary: at least one of both propositions is completely possible.

The necessity measure traduces the inclusion degree between the set of possible values of F with the compatibles values of Ref. This indicator completes the possibility measure by indicating the degree with which the information is certain.

As an example, consider Figure 6 (that follows the example of Figure 5). It is possible that the occurrence of event E is small ($\Pi_{E \in small} = 0,5$) but there is no certitude on it ($\mathrm{N}_{E \in small} = 0$).

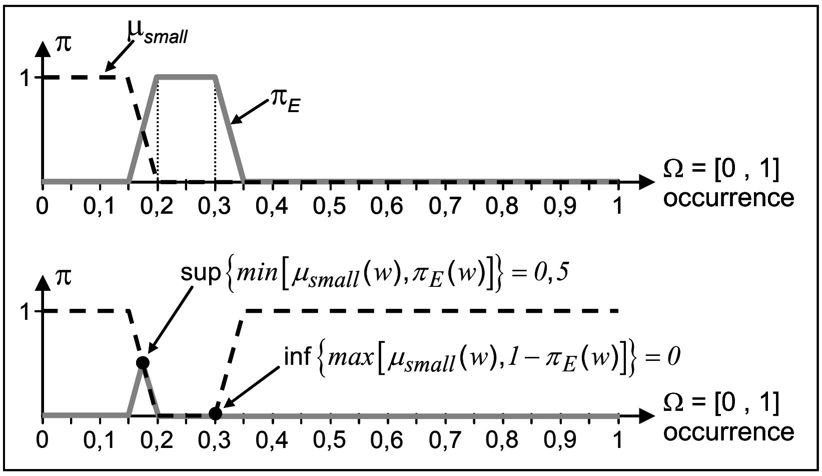

Figure 6. Illustration of possibility and necessity measures

Let finally introduce an interesting characteristics of possibility and necessity measures with regard to probability. It can be shown that an equivalence class $\mathcal{P}$ of probabilities measures Pr for an event A can be defined as:

$$\mathcal{P} = \left\{ Pr / \forall A, \mathrm{N}(A) \leq Pr(A) \leq \Pi(A) \right\} \tag{12}$$

3.2.2 Principle of the fuzzy / possibilistic reliability modeling approach

Let assume that both the degradation signal and the degradation limit are modeled with possibility distributions. At time t, the degradation signal y has a possibility distribution $\pi_{y/t}$ that designates the possible degradation state of the system, and the possibility distribution $\mu_{y_{lim}/t}$ represents the membership function of failure, e.g. the set of degradation value that are compatible with the event "failure" (Figure 7).

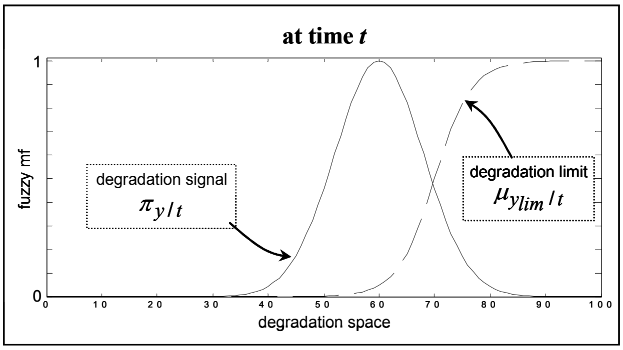

Figure 7. Possibility distributions of the degradation signal and limit at time *t*

According to eq. (11), possibility and necessity indicators enable evaluating the degree with which the assertion "the degradation signal is within the failure member set" and the degree of certainty of this assertion as follows. At time t,

$$\begin{aligned} \Pi_{degr \in fail}(t) &= \sup_{u \in Y}\left\{min\left[\mu_{y_{lim}/t}(u), \pi_{y/t}(u)\right]\right\} \\ \mathrm{N}_{degr \in fail}(t) &= \inf_{u \in Y}\left\{max\left[\mu_{y_{lim}/t}(u), 1 - \pi_{y/t}(u)\right]\right\} \end{aligned} \tag{13}$$

According to eq. (12), possibility and necessity indicators enable bounding the probability of occurrence of an event. Thereby,

$$\mathrm{N}_{degr \in fail}(t) \le Pr\left[failure\right](t) \le \Pi_{degr \in fail}(t) \tag{14}$$

Following that, the reliability modeling expression is at time t:

$$\begin{aligned} 1 - \Pi_{degr \in fail}(t) &\le 1 - Pr\left[failure\right](t) \le 1 - \mathrm{N}_{degr \in fail}(t) \\ 1 - \Pi_{degr \in fail}(t) &\le R(t) \le 1 - \mathrm{N}_{degr \in fail}(t) \end{aligned} \tag{15}$$

3.2.3 Illusration and discussion

An illustration of the fuzzy / possibilistic approach is proposed in Figure 8 in which the 3D-graph is that of the possibilistic modeling of the degradation signal an limit, and the 2D-graph, that of the resulting failure and reliability modeling.

In practice, this approach can be used if both the degradation signal and the degradation limit are expressed as possibility distributions functions. Thus, when performing the prediction step of prognostic, this implies that one must be able to fuzzyfy the degradation signal estimation. In regard to the degradation limit, fuzzy modeling is particularly adapted to the formalization of expert knowledge. At this stage, note that one can directly construct the set of non acceptable degradation values without looking for a defined threshold: indeed, in Figure 8, $\mu_{y_{lim}}(t,y)$ can be seen as a "cumulative distribution function". Moreover, if sufficient experiences are available, the degradation limit membership functions can be obtained (like with statistical approach) by using neuro-fuzzy system as proposed by [4].

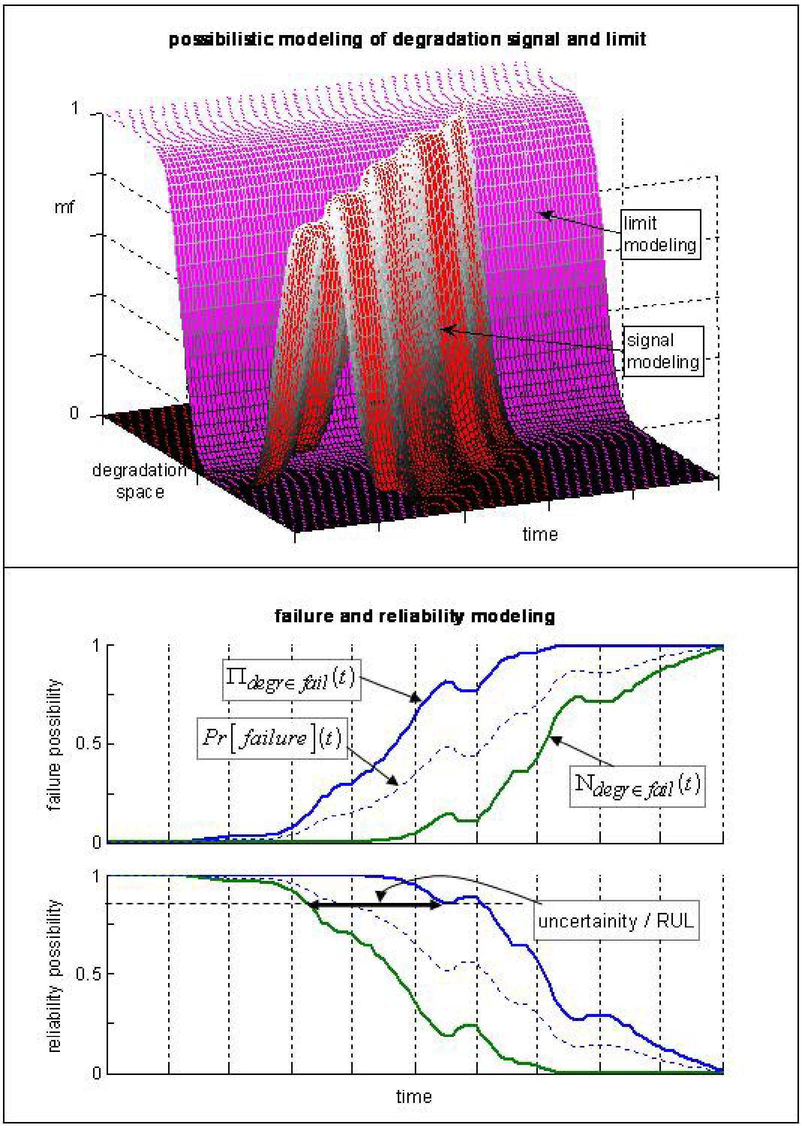

Figure 8. Illustration of the possibilistic approach of reliability modeling.

According to eq. (15), possibility theory enables giving boundaries to the reliability modeling and, thereby, allows estimating more confidently the remaining useful life or time to failure of a system (as proposed in Figure 1). In opposite to it, probability theory does not "conserve" the uncertainty of knowledge since the process of reliability modeling results in a single aggregated indicator: the same confidence will be accorded to two situations for which the formalized knowledge can be very different (spreading of the degradation signal and limit). Following that, fuzzy / possibilistic approach of reliability modeling enables practitioners to be more critic on the risk incurred by the system and therefore, to build adequate maintenance policies. Let also note that the approach is much more computationally effective than that of probability theory since it is based on "min, max, sup, inf" operators.

4. Conclusion

In maintenance field, traditional concepts like preventive and corrective strategies are progressively completed by new ones like predictive and proactive maintenance. For that purpose, a fundamental task is the estimation of the provisional reliability of equipment as well as its remaining useful life. In the paper, reliability modeling is assimilated to the prognostic process since two mains tasks must be ensured: a first one to predict the evolution of the degradation of the system, and a second one to asses this predicted situation with regard to a degradation limit referential. Following that, two ways of taking into account the uncertainty are discussed.
The traditional approach of reliability based on statistical analysis can be not suitable as very few knowledge can be available: real systems are complex; there are many uncertainties upon their behaviors. Moreover, it can be difficult and computationally time expensive to deploy if real systems are complex and non-linear.
The fuzzy / possibilistic approach of on-line reliability modeling and estimation is well adapted to the integration of expertise. Moreover, such an approach aims at considering the available knowledge as it is: uncertain and imprecise if necessary. As a consequence, the processed reliability indicators can be considered with more confidence. In addition, this approach is not so time expensive. However, it requires the fuzzyfication of the degradation signal prediction.

The work reported here is still a prospective one and it is obviously extended. The main developments that are at present led deal with the reliability modeling of components with various failure modes and to the extension of the approach to a global system (composed of different components whose reliability characteristics can be fuzzily estimated). In such cases, although there is a great interest in considering knowledge as something imprecise and uncertain, global treatments can led to situation in which there is no way to conclude upon adequate maintenance strategies since individual reliabilities are expressed as intervals…

References

[1] CEN/TC 319 (2001). *Maintenance terminology*. European Standard, European Committee for Standardization.

[2] ISO 13381-1 (2004). *Condition monitoring and diagnostics of machines - prognostics - Part1: General guidelines*. International Standard, ISO.

[3] Lu, S., H. Lu and W.J. Kolarik (2001). Multivariate performance reliability prediction in real-time, *Reliability Eng. and System Safety*, vol. 72, pp. 39-45.

[4] Chinnam, R. and B. Pundarikaksha (2004). A neurofuzzy approach for estimating mean residual life in condtition-based maintenace systems. *Int. J. materials and Product Technology*, vol. 20:1-3, pp. 166–179.

[5] Wang, P. and D. Coit (2004). Reliability prediction based on degradation modeling for systems with multiple degradation measures. *In: Proc. of Reliab. and Maintain. Ann. Symp. - RAMS*, pp. 302–307.

[6] Jardine, A., D. Lin, and D. Banjevic (2006). A review on machinery diagnostics and prognostics implementing condition-based maintenance. *Mech. Syst. and Sign. Proc*, vol. 20, pp. 1483–1510.

[7] Dragomir, O., R. Gouriveau and N. Zerhouni (2007). Framework for a distributed and hybrid prognostic system. *In: 4th IFAC Conf. on Management and Control of Production and Logistics, MCPL 2007, Sibiu, Romania*.
[8] Zadeh, L. (1978). Fuzzy Sets as the Basis for a Theory of Possibility, *Fuzzy Sets and Systems*, vol. 1, pp. 3-28.
[9] Zadeh, L. (1965). Fuzzy sets. *Information and Control*, vol. 8:33, pp. 338-353.
[10] Dubois, D. and H. Prade (1988). *Possibility Theory - An Approach to Computerized Processing of Uncertainty*. New York, Plenum Press.